වයඹ පළාත් අධ්‍යාපන දෙපාර්තමේන්තුව

Provincial Department of Education - NWP

10	E	II

Third Term Test - Grade 13 - 2016

Index No : ..... Combined Mathematics II Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
* **Part B**
  Answer five questions only. Write your answers on the sheets provided.
* *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
* *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics II		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper I total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

1. A train normally travels with a constant velocity U. In a certain day, during the motion, due to a construction of road the train comes to rest with constant retardation f and in the same instant it starts to travel with constant acceleration of $2f$ and obtains the velocity U again. Use the velocity time graph to show that the loss of time of the train, due to the construction of the road, is $\frac{3U}{4f}$.

2. A particle travels as a projectile. The velocity of the particle in the maximum height is $\sqrt{\frac{2}{3}}$ times than the velocity of the half of the maximum height. Show that the angle of projection is $\frac{\pi}{4}$

3. A sphere collides directly with another sphere with mass, m times than the mass of the first sphere and the velocity, $\frac{1}{n}$ times than the velocity of the first sphere. If the first sphere comes to the rest due to the collision, show that the coefficient of elasticity between the two spheres is $\frac{(m+n)}{(mn-n)}$.

4. An engine working at constant power 500 kw, pulls a train along an inclined track of inclination 1 in 196. The total mass of the train with the engine is 2.5×10^5 kg. When the velocity of it is 24 kmh^{-1}, the acceleration is 0.2 ms^{-2}. Find the constant resistance in newton, against the motion of the train ($g = 9.8\ ms^{-2}$).

5. $\underline{a}$ and $\underline{b}$ are two non zero vectors. Two vectors $\underline{a}$ and $(\underline{a}+\underline{b})$ are perpendicular to each other. If $\underline{b} = 2\underline{a}$, Show that $\underline{b}$ is perpendicular to $4\underline{a} + \underline{b}$.

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

6. A weighted uniform ladder is in limiting equilibrium, so that one end is in contact with a smooth vertical wall and the other end is touching the rough horizontal floor, making an angle θ with the vertical. If the angle of friction is λ, show that $2\tan\lambda = \tan\theta$.

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

7. A and B are two events such that $P(A) = \frac{3}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cup B) = \frac{7}{10}$. Find

   i. $P(A \cap B)$

   ii. $P(A \mid B')$

   iii. Are A and B independent?

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

8. In a shooting event, for the three players A, B, and C, the probabilities of shooting the target correctly, are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively.

   Each player is firing a shot and find the probability of hitting the target correctly by one shot.

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

9. Out of *n* numbers, there are **p** number of one's and the remaining are q number of zeros. Show that the standard deviation of these numbers is $\frac{\sqrt{pq}}{n}$.

10. An incomplete frequency distribution is given below.

x	10-20	20-30	30-40	40-50	50-60	60-70	70-80
f	12	30	f_1	65	f_2	25	19

If it is given that $\sum f = 230$ and the median is 46, Find the values of f_1 and f_2.

Combined Mathematics 13 -II (Part B)

11.

a) A small sphere is kept on the smooth horizontal floor in between two vertical parallel walls, distance "a" apart. The sphere is projected form foot of a wall towards the other wall with velocity U perpendicular to the wall. The coefficient of elasticity between the sphere and each wall is e. 0<e<1.

i. Draw a rough diagram of the velocity time graph for the motion of the sphere up to the third collision.

ii. Hence show that the time taken for the sphere up to the third collision is $\frac{a}{u}\left[1+\frac{1}{e}+\frac{1}{e^2}\right]$.

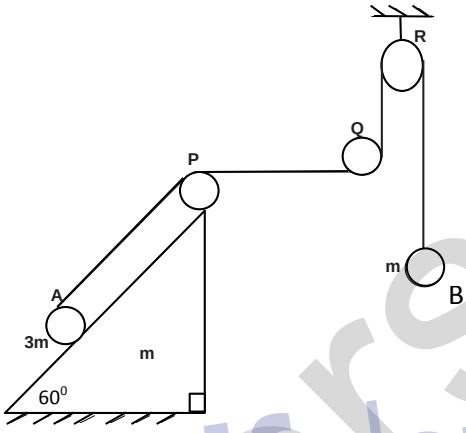
iii. Show that the total time taken for the sphere up to the n^{th} collision is $\frac{a}{u}\left(\frac{e^n-1}{e^n-e^{n-1}}\right)$.

b) A river with straight parallel banks and breadth "a" flows at an uniform speed u . A and B are two points on opposite banks such that the point B is below the point A. The line AB makes an angle $\boldsymbol{\alpha}$ with the river banks. Two boats X and Y moving with velocities v and w (v, $w > u$) in still water moves along the line, AB starting from the points A and B respectively. Draw velocity triangles for the journey of x and y and show that the time taken for the two boats to meet each other is $\frac{a\ cosec \propto}{\sqrt{v^2-u^2\sin^2\propto}+\sqrt{w^2-u^2\sin^2\propto}}$.
Find the distance to the interception point from the point A.

12.

a) A particle *P* of mass m suspends freely from a point O using a light inextensible string of length $2a$. When it is in equilibrium position, a velocity of $\sqrt{5ag}$ is given horizontally. Find the velocity of the particle and the tension in the string when the string makes an angle 60^0 with the downward vertical. Now the string hits a peg Q which is $\frac{a}{2}$ vertically below from the level of the point O. In the subsequent motion, show that the particle comes to instantaneous rest when P is vertically above Q.

b)



Two particles A and B with masses **3m** and **m** are attached to the two ends of a light inextensible string such that A is on the face inclined at an angle of 60^0 to the horizontal of a wedge which is movable along a horizontal table and B hangs freely. The string passes through the pulleys P, Q and R . The part PQ of the string is horizontal and the parts QR and RB of the string are vertical. The system is released from rest, find the accelerations of the particles A,B and the wedge.

13. One end of an elastic string with natural length $\boldsymbol{a}$ and modulus of elasticity **mg** is attached to a point **O**, which is at a distance $\mathbf{2}\boldsymbol{a}$ from the edge of a smooth horizontal table. The other end of the string is attached to a particle **P** of mass **m**. The particle **P** is attached to one end of a light inextensible string passes through the edge of the table and a particle **Q** of mass **m** is hung by the other end of the string. The particle **P** is kept at **O** such that it is in a vertical plane perpendicular to the edge of the table and released from rest. Let the length of the elastic string after time **t** is x, Show that $\ddot{x} + \frac{g}{2a}(x - 2a) = 0$. Where $a < x < 2a$. Let, x - $2a$ = A Cos ωt + B Sin ωt, Find the values of A, B and ω.

Show that the particle P reaches the edge of the table after time $\pi\sqrt{\frac{a}{2g}}$ with a velocity of $\sqrt{\frac{ag}{2}}$.

Find the tension in the string at any time such that, $o < t < \pi\sqrt{\frac{a}{2g}}$.

14.

a) The position vectors of the points A and B relative to a point O are $\underline{a}$ and $\underline{b}$. Find the position vector of any point P on the line AB. Show that $\underline{P} = \underline{a} + \lambda\left(\underline{b} - \underline{a}\right)$. Where λ is a parameter.

The position vectors of four points A,B,C and D relative to the point O are $\underline{a}$, $\underline{b}$, $\underline{2b}$ and $\frac{1}{3}\ \underline{a}$ respectively. The lines AB and CD intersect at E. Find the position vector of E.

b) The length of a side of a regular hexagon ABCDEF is $2a$. The forces of magnitudes P,2P,Q,Q, 2P and P act along the sides AB, BC, CD, DE, EF and FA respectively.

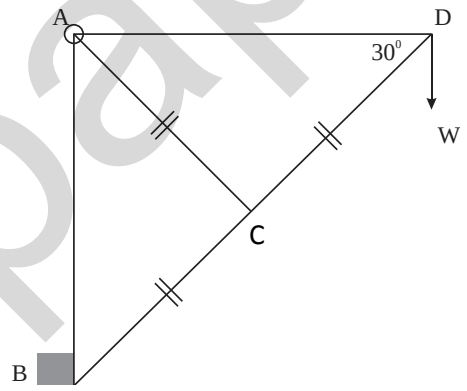
If the resultant of the system of forces passes through the point E, show that it passes through C also. Find the magnitude of the resultant. A couple G act in the same plane is applied in the anticlockwise direction to the system of forces. If $G = 8\sqrt{3}\ Pa$, find the distance to the point which the resultant cut the line AB from A.

15. a) The lengths of three uniform rods AB,BC and CD are, l_1, l_2 and l_3 respectively and their weights are proportional to the lengths of the rods. These three rods are jointed freely at B and C and the system is kept horizontally on two smooth pegs at P and Q. Show that the distance between two pegs P and Q is,

$$PQ = \frac{l_1^2}{2l_1 + l_2} + \frac{l_3^2}{2l_3 + l_2} + l_2$$

Find the reactions at P and Q. Also find the reaction on B at the joint B.

b)



The figure represent a framework made by joining five light rods, which is hinged to a smooth vertical wall at A and is in equilibrium using a weight W applied at D. Find the reactions at A and B. Draw a stress diagram and hence find the stresses on rods distinguishing tensions and thrusts.

16.

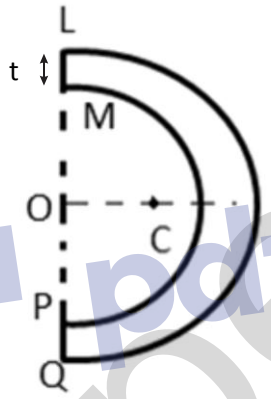
a) Show that the centre of gravity of an uniform solid hemisphere of radius r lies on the axis of symmetry at a distance $\frac{3r}{8}$ from the centre O of the hemisphere.

A bowl is made by removing a solid hemisphere of radius r- t, from the solid hemisphere of radius r and centre O. If the centre of gravity of the bowl is C, Show that $oc = \frac{3r}{8}\left\{\frac{1-(1-\alpha)^4}{1-(1-\alpha)^3}\right\}$ where $\alpha = \frac{t}{r}$.

If $t = \frac{r}{10}$, find the value of $\frac{oc}{r}$.

When α is very small, show that $1-(1-\alpha)^4 = 1-4\alpha$ and $1-(1-\alpha)^3 = 1-3\alpha$.

Hence; when the above body is a hollow hemisphere, show that OC $= \frac{r}{2}$.



The bowl is hung freely from the point L. If it is in the equilibrium find the inclination of LQ to the vertical.

17.

a) There are three book racks in a book shelf in a library of a school. They are labelled as A,B and C. There are both chemistry and physics books in all the three racks.

	Chemistry	Physics
A	15	10
B	17	08
C	35	65

i. If Janaka select a book from this book shelf randomly, find the probability that the selected book is,
   a. a chemistry book.
   b. a physics book.

ii. First Janaka select a rack randomly from the book shelf and then select a book. Find the probability that the selected book is a chemistry book.

iii. If the selected book by Janaka is a chemistry book, find the probability that it was selected from rack A.

b) The runs obtained by two batsmen Mahela and Lasith in 10 cricket matches are given below. Out of them find the batsman who gets runs very well and find the most stable batsman.

Mahela	17,	111,	9,	67,	13,	22,	104,	37,	79,	41
Lasith	49,	12,	77,	37,	06,	51,	39,	52,	17,	20